

# Coulomb Distortion Effects for $(e, e'p)$ Reactions at High Electron Energy

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## Abstract

We report a significant improvement of an approximate method of including electron Coulomb distortion in electron induced reactions at momentum transfers greater than the inverse of the size of the target nucleus. In particular, we have found a new parametrization for the elastic electron scattering phase shifts that works well at all electron energies greater than  $300\text{ MeV}$ . As an illustration, we apply the improved approximation to the  $(e, e'p)$  reaction from medium and heavy nuclei. We use a relativistic “single particle” model for  $(e, e'p)$  as applied to  $^{208}\text{Pb}(e, e'p)$  and to recently measured data at CEBAF on  $^{16}\text{O}(e, e'p)$  to investigate Coulomb distortion effects while examining the physics of the reaction.

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Electron scattering has long been acknowledged as a useful tool for investigating nuclear structure and nuclear properties, especially in the quasielastic region. One of the primary attributes of electron scattering as usually presented is the fact that in the electron plane-wave Born approximation, the cross section can be written as a sum of terms each with a characteristic dependence on electron kinematics and containing various bi-linear products of the Fourier transform of charge and current matrix elements. That is, various structure functions for the process can be extracted from the measured data by so-called Rosenbluth separation methods. The trouble with this picture is that when the Coulomb distortion of the electron wavefunctions arising from the static Coulomb field of the target nucleus is included exactly by partial wave methods, the structure functions can no longer be extracted from the cross section, even in principle.

Electron Coulomb distortion in elastic and inelastic scattering for various processes has been included with various approximations in the past [1–5]. In the early 90’s Coulomb distortion for the reactions  $(e, e')$  and  $(e, e'p)$  in quasielastic kinematics was treated exactly by the Ohio University group [6–10] using partial wave wave expansions of the electron wave functions. Such partial wave treatments are referred to as the distorted wave Born approximation (DWBA) since the static Coulomb distortion is included exactly by numerically solving the radial Dirac equation containing the Coulomb potential for a finite nuclear charge distribution to obtain the distorted electron wave functions. The induced transition by a virtual photon is included to first order (the Born Approximation). While this calculation permits the comparison of various nuclear models to measured cross sections and provides an invaluable check on various approximate techniques of including Coulomb distortion effects, it is numerically challenging and computation time increases rapidly with higher incident electron energy. And, as noted above, it is not possible to separate the cross section into various terms containing the structure functions and develop insights into the role of various terms in the charge and current distributions.

In all of our DWBA investigations of  $(e, e')$  and  $(e, e'p)$  reactions in the quasielastic region, we used a relativistic treatment based on the  $\sigma - \omega$  model for the nucleons involved.

In particular, for the  $(e, e'p)$  reaction we use a relativistic Hartree single particle model for a bound state [11] and a relativistic optical model for an outgoing proton [12] combined with the free space relativistic current operator

$$J^\mu = \gamma^\mu + i \frac{\kappa}{2M} \sigma^{\mu\nu} \partial_\nu. \quad (1)$$

Using this model, we compared our DWBA calculations with experimental data measured at various laboratories for  $(e, e')$  [6,7], and for  $(e, e'p)$  [8–10] and have found excellent agreement with the data. We concluded that the relativistic nuclear models are in excellent agreement with the measured data and do not need to invoke meson exchange effects and other two-body terms in the current that are necessary in a Schrödinger description that uses a non-relativistic reduction of the free current operator [13]. Therefore, in this brief report we will continue to use our relativistic “single-particle” model to investigate Coulomb distortion effects and to compare to the newly measured data from CEBAF.

To avoid the numerical difficulties associated with DWBA analyses at higher electron energies and to look for a way to still define structure functions, our group [14,15] developed an approximate treatment of the Coulomb distortion based on the work of Knoll [16] and the work of Lenz and Rosenfelder [17]. Knoll examined approximations of the Green function valid for large momentum transfers (that is, valid for  $qR > 1$  where  $R$  is the size of the target) while Lenz and Rosenfelder constructed plane-wave-like electron wavefunctions which included Coulomb distortion effects. We were able to greatly improve some previous attempts along this line [18,19] where various additional approximations were made which turned out not to be valid. We did have the advantage of having the exact DWBA calculation available for incident electron energies up to  $400 - 500 MeV$  for checking our approximations. We compared our approximate treatment of Coulomb distortion to the exact DWBA results for the reaction  $(e, e'p)$  and found good agreement (at about the 1-2% level) near the peaks of cross sections even for heavy nuclei such as  $^{208}Pb$ . The agreement was not so good away from the peaks.

As discussed in our previous papers [14,15], one of the ingredients of our approximate

electron wavefunction is a parameterization of the elastic scattering phase shifts in terms of the angular momentum. In this paper, we briefly review our previous approximation of the Coulomb distorted electron wavefunction and present a greatly improved parametrization of the phase shifts which works well at all incident electron energies greater than 300 *MeV*. In addition, we will compare our relativistic “single-particle” model to new experimental data from CEBAF.

Our approximate method of including the static Coulomb distortion in the electron wavefunctions is to write the wave functions in a plane-wave-like form [15];

$$\Psi^\pm(\mathbf{r}) = \frac{p'(r)}{p} e^{\pm i\delta(\mathbf{L}^2)} e^{i\Delta} e^{i\mathbf{p}'(r)\cdot\mathbf{r}} u_p, \quad (2)$$

where the phase factor  $\delta(\mathbf{L}^2)$  is a function of the square of the orbital angular momentum operator,  $u_p$  denotes the Dirac spinor, and the local effective momentum  $\mathbf{p}'(\mathbf{r})$  is given in terms of the Coulomb potential of the target nucleus by

$$\mathbf{p}'(\mathbf{r}) = \left( p - \frac{1}{r} \int_0^r V(r) dr \right) \hat{\mathbf{p}}. \quad (3)$$

We refer to this  $r$ -dependent momentum as the Local Electron Momentum Approximation (LEMA). The *ad-hoc* term  $\Delta = a[\hat{\mathbf{p}}'(r)\cdot\hat{\mathbf{r}}]\mathbf{L}^2$  denotes a small higher order correction to the electron wave number which we have written in terms of the parameter  $a = -\alpha Z(\frac{16\text{MeV}/c}{p})^2$ . The value of 16 MeV/c was determined by comparison with the exact radial wave functions in a partial wave expansion.

The elastic scattering phase shifts are labelled by the Dirac quantum number  $\kappa$  which takes on plus minus integer values beginning with one. The eigenvalues of  $\mathbf{J}^2$  are  $j(j+1)$  which equals  $\kappa^2 - \frac{1}{4}$ . The basic idea of our approximation is to calculate the elastic scattering phases and fit them to function of  $\kappa^2$ . Then to replace the discrete values of  $\kappa^2$  with the total angular momentum operator  $\mathbf{J}^2$  which we then replace by the orbital angular momentum operator  $\mathbf{L}^2$  since the low  $\kappa$  terms where the difference between  $j$  and  $l$  is significant contribute very little to the cross section. The removal of any spin dependence apart from what is in the Dirac spinor  $u_p$  is crucial for defining modified structure functions.

Based on earlier work by others we fitted the elastic scattering phases shifts to a power series in  $\kappa^2$  up to second order;

$$\delta_\kappa = b_0 + b_2\kappa^2 + b_4\kappa^4, \quad (4)$$

where the coefficients,  $b_0$ ,  $b_2$ ,  $b_4$  are extracted from a best fit for  $\kappa$  values up to about  $3pR$  where  $R$  is the nuclear radius. Note that this procedure requires calculating the elastic scattering phase shifts for the incident and outgoing electron energies up to  $\kappa$  values of order  $3pR$ , which for high electron energies can be quite demanding computationally. We refer to these phases as the  $\kappa^2$ -dependence phases. This fit to the phases worked very well for  $\kappa$  values up to approximately  $\kappa = 3pR \approx 35$  at medium or low energy, but did not fit the exact phases shifts very well for higher energies where  $\kappa = 3pR \geq 50$  or more. Since we were primarily looking at electron energies in the 300-600 MeV range in our previous work, this discrepancy did not present a significant problem.

However, with CEBAF type energies we need a fit to the phases that will work at any incident energy where the overall approximation can be used; that is, for incident electron energies greater than about 300 MeV and processes with momentum transfers greater than about  $1/R$ . In addition, we would like to avoid calculating all of the elastic phase shifts, particularly the very high ones. A reasonable solution is to make use of the fact that the higher  $\kappa$  phase shifts approach the point Coulomb phases which have a simple analytical form at high energy. At the other extreme, the low  $\kappa$  phases corresponding to orbitals which penetrate the nucleus are linear in  $\kappa^2$  which was the basis of our initial parametrization. The difficult phases to fit correspond to  $\kappa$  values of order  $pR$  which, from a classical point of view, corresponds to scattering from the nuclear surface. Moreover, it is well known that in electron induced reactions the spatial region around the surface gives the largest contribution to the cross section, so it is important to fit the intermediate range as well as possible.

Another goal is to reduce the computer time needed, so we decided to seek a parametrization of the elastic scattering phases shifts in terms of  $\kappa^2$  which has the correct large  $\kappa^2$

behaviour and becomes linear in  $\kappa^2$  at low angular momentum. Since we have the correct large  $\kappa$  behaviour, we need only calculate the exact scattering phase shifts for  $\kappa$  values of up to of order  $pr$ . The large  $\kappa$  and small  $\kappa$  behaviour are quite different, so we chose to write the expression for the phase shift as the sum of two terms with an exponential factor which suppresses one of the terms at small  $\kappa$  values and the other at large  $\kappa$  values. After some experimentation, we find that the following parametrization of elastic scattering phase shift describes the exact phase shifts very well:

$$\delta(\kappa) = [a_0 + a_2 \frac{\kappa^2}{(pR)^2}] e^{-\frac{1.4\kappa^2}{(pR)^2}} - \frac{\alpha Z}{2} (1 - e^{-\frac{\kappa^2}{(pR)^2}}) \times \ln(1 + \kappa^2) , \quad (5)$$

where  $p$  is the electron momentum and we take the nuclear radius to be given by  $R = 1.2A^{1/3} - 0.86/A^{1/3}$ . We fit the two constants  $a_0$  and  $a_2$  to two of the elastic scattering phase shifts ( $\kappa = 1$  and  $\kappa = \text{Int}(pR) + 5$ ). To a very good approximation,  $a_0 = \delta(1)$  and  $a_2 = 4\delta(\text{Int}(pR) + 5) + \alpha Z \ln(2pR)$ , where  $\text{Int}(pR)$  replaces  $pR$  by the integer just less than  $pR$ . Note that this parametrization only requires the value of the exact scattering phase shift for  $\kappa = 1$  and  $\kappa = \text{Int}(pR) + 5$ . As shown in Fig. 1, the  $\kappa^2$ -dependence phase parametrization breaks down for high  $\kappa$  values and has large deviations for mid-range  $\kappa$  values. The new phase parametrization fits the exact phases very well for electron energy of  $E = 2400$  MeV on  $^{16}\text{O}$ , although the new phase parametrization does still show some small deviations from the exact phases for  $\kappa$  values around 20 to 30 which is in the surface region. Clearly additional terms could be added to the parametrization to obtain a better fit. However, as we shall see below, the simple fit that we have used reproduces the cross section quite well.

Using the new phase shift parametrization and the local effective momentum approximation we construct plane-wave-like wave functions for the incoming and outgoing electrons. Since the only spinor dependence is in the Dirac spinor all of the Dirac algebra goes through as usual and we end up with a Møller-like potential which contains an  $r$ -dependent momentum transfer. It is then straightforward to calculate the  $(e, e'p)$  cross sections and modified

structure functions. Please see our previous papers [14,15] for details.

In most  $(e, e'p)$  experiments, there is sufficient energy resolution that protons knocked out of different shells can be examined. It is common to report the experimental results in terms of the reduced cross section  $\rho_m$  as a function of missing momentum  $p_m$ , which is defined by

$$\rho_m(p_m) = \frac{1}{PE_p\sigma_{eP}} \frac{d^3\sigma}{dE_f d\Omega_f d\Omega_P} , \quad (6)$$

where the missing momentum is determined by the kinematics  $\mathbf{p}_m = \mathbf{P} - \mathbf{q}$  where  $\mathbf{P}$  is the outgoing proton momentum and  $\mathbf{q}$  is the asymptotic momentum transfer from the electron defined by  $\mathbf{q} = \mathbf{p}_i - \mathbf{p}_f$ . For plane wave protons in the final state  $\rho_m$  is related to the probability that a bound proton from a given shell have momentum  $\mathbf{p}_m$ . For the off-shell electron-proton cross section  $\sigma_{eP}$  we use the form ‘cc1’ given by de Forest [20]. For distorted outgoing protons, this reduced cross section is just a convenient way of comparing experiment and theory since the theory results for the cross section can have the same factors removed. Note that all calculations will be carried out in the laboratory system (target fixed frame).

While there are two experimental kinematic arrangements commonly used in  $(e, e'p)$  experiments with designations of parallel kinematics and perpendicular kinematics, in the present work, we consider only perpendicular kinematics. In perpendicular kinematics, the momentum transfer  $\mathbf{q}$  is held fixed along with the magnitude of the momentum of the outgoing proton while the angle between  $\mathbf{q}$  and  $\mathbf{P}$  is varied. The calculated reduced cross section is compared (by means of a linear least squares fit) to the similarly reduced experimental cross section to extract an overall scale factor which is the spectroscopic factor. The spectroscopic factor contains two factors, the occupation probability of a proton in a given orbit and the overlap of the residual nucleus with the  $A - 1$  nucleons in the target.

As a test case, we calculate the reduced cross sections with the new phases for a heavy nucleus,  $^{208}\text{Pb}$ . Figure 2 shows the reduced cross section as a function of the missing momentum  $p_m$  for knocking protons from the  $3s_{1/2}$  orbital of  $^{208}\text{Pb}$ . The incident electron energy  $E_i = 412$  MeV, and the outgoing proton kinetic energy is  $T_p = 100$  MeV. We have

chosen  $P = q$  which corresponds to an electron scattering angle of  $\theta_e = 74^\circ$ . The solid line is the result of the full DWBA [8], the dashed curve is the result with the new phase shift parametrization, and the dotted curve is the result with  $\kappa^2$ -dependence phase shift parametrization. The dashed curve obtained by using the new phases clearly reproduces the exact result much better than the previous  $\kappa^2$ -dependence phase parametrization over the whole region.

We also apply the new phase shift parametrization for the case of high energy electron scattering on the light nucleus  $^{16}\text{O}$  where protons are removed from the  $p_{1/2}$  and  $p_{3/2}$  orbits. The incident electron energy  $E_i = 2441.6$  MeV and the outgoing proton kinetic energy  $T_p = 427$  MeV as shown Fig. 3. In this figure, the solid curves are the approximate DWBA results using the new phase shift parametrization, the dotted curves are the PWBA results without Coulomb distortion, and the data are newly measured from CEBAF as reported in the dissertation of Gao [21]. Note that our exact DWBA code cannot evaluate such high energy processes without extensive modification which we have not done.

As expected, the effect of Coulomb distortion on such a high energy electron induced process is very small except possibly at large missing momentum. Note that the Coulomb effects for  $^{16}\text{O}$  in the medium energy region (500 MeV) was of the order of 3% [10]. This fit to the experimental data using our relativistic “single particle” model for the nucleon wavefunctions results in spectroscopic factors of 61% for the  $p_{1/2}$  orbital and 70% for the  $p_{3/2}$  orbital. In our analysis of Saclay data [23] at lower electron energies using a similar nuclear model we found spectroscopic factors of 54% and 57% respectively [10].

In summary, we have improved our previous approximate method of including Coulomb distortion effects in  $(e, e'p)$  reactions from nuclei. The improvement involves a better parametrization of the elastic scattering phase shifts which has the correct behaviour for large angular momenta and requires the calculation of only two phase shifts (for  $\kappa = 1$ , and for  $\kappa$  equal to  $\text{Int}(pR) + 5$ ). We showed that even for  $(e, e'p)$  on  $^{208}\text{Pb}$  the cross section calculated with our approximation using the improved parametrization of the phase shifts agrees with the exact DWBA result quite well even out beyond the second maxima.



This is a significant improvement over our previous approximation for the phase shifts. In addition, we compared our relativistic “single-particle” model for  $(e, e'p)$  from  $^{16}\text{O}$  to the recently measured cross section at Thomas Jefferson Lab and found excellent agreement for the removal of a proton from the  $p_{3/2}$  and  $p_{1/2}$  shells with reasonable spectroscopic factors.

Our improved approximate method of including Coulomb distortion in electron scattering reactions works for high energy electrons as well as for more moderate energies (300 – 500 MeV), and for experiments at the few percent level this approximate way of including Coulomb distortion is adequate. More importantly, as discussed in our previous paper, this “plane-wave-like” approximation permits the extraction of “structure functions” even in the presence of strong Coulomb effects and thus provides a very good tool for looking into the response of the nucleus to “longitudinal” and “transverse” photons.

# FIGURES

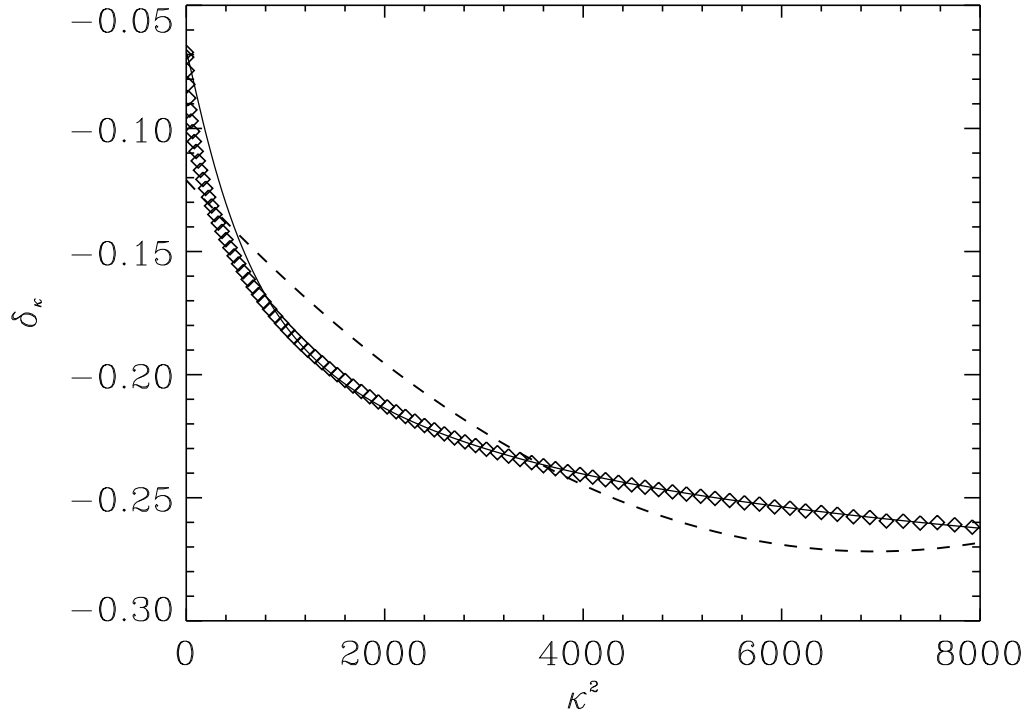


FIG. 1. The comparison between the exact,  $\kappa^2$ -dependent, and new phases in  $^{16}\text{O}$  for  $\kappa_{max} = 100$  and energy  $E = 2441$  MeV. The diamonds are the exact phases, the dashed curve for  $\kappa^2$  fit, and the solid curve for the new phase shift parametrization.

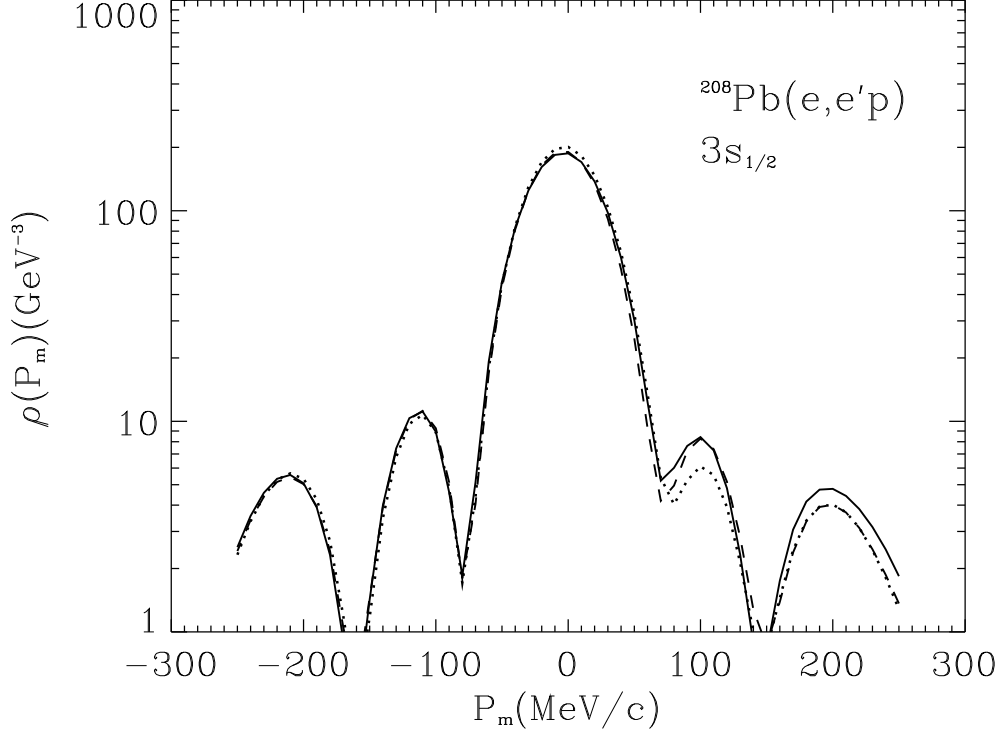


FIG. 2. The reduced cross section for  $^{208}\text{Pb}$  for the  $3s_{1/2}$  orbit with perpendicular kinematics. The kinematics are  $E_i = 412$  MeV, and proton kinetic energy  $T_P = 100$  MeV. The solid line is the full DWBA result, the dashed line is the approximate DWBA using the new phase shift parametrization, and the dotted line is the approximate DWBA with the  $\kappa^2$ -dependent phases.

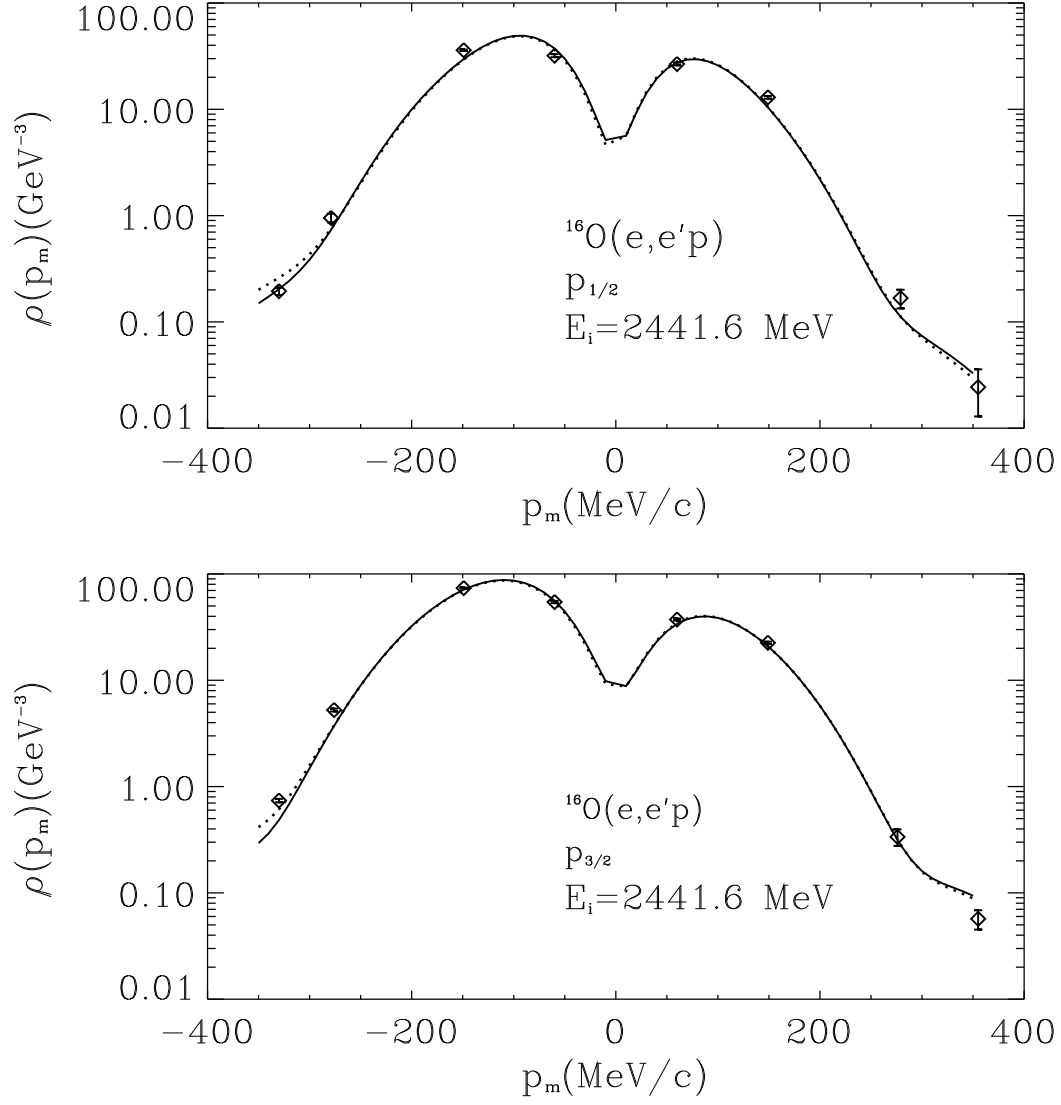


FIG. 3. The cross sections for  $^{16}\text{O}$  from  $p_{1/2}$  and  $p_{3/2}$  orbits for perpendicular kinematics. The kinematics are  $E_i = 2441.6$  MeV, proton kinetic energy  $T_P = 427$  MeV, and energy transfer  $\omega = 436$  MeV. The solid lines are the new approximate DWBA results, the dotted lines are the PWBA results, and the data are from CEBAF [21].

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